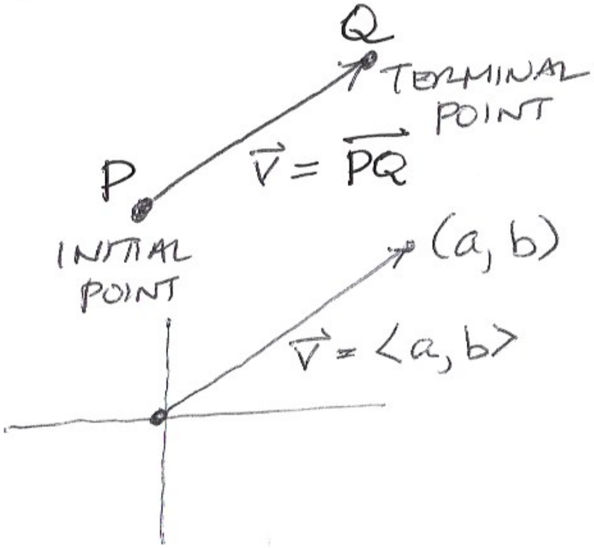
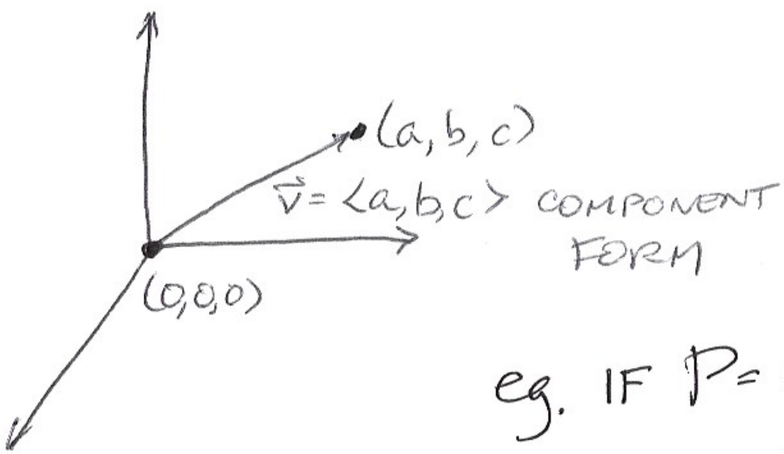
11.2 3D VECTORS



A VECTOR WITH INITIAL POINT (x_0, y_0, z_0) AND TERMINAL POINT (x_1, y_1, z_1) HAS COMPONENT FORM

$$\langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle$$

(IE. "TERMINAL MINUS INITIAL")



eg. IF $P = (-4, 2, -1)$ AND $Q = (-7, 5, 2)$

THEN $\overrightarrow{PQ} = \langle -7 - -4, 5 - 2, 2 - -1 \rangle$

$= \langle -3, 3, 3 \rangle$

$$\| \langle v_1, v_2, v_3 \rangle \| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

MAGNITUDE (OR LENGTH) OF VECTOR $\langle v_1, v_2, v_3 \rangle$

eg. $\|\overrightarrow{PQ}\| = \sqrt{(-3)^2 + 3^2 + 3^2}$

$= \sqrt{3^2((-1)^2 + 1^2 + 1^2)} = 3\sqrt{3}$

A UNIT VECTOR IS A VECTOR OF MAGNITUDE 1

eg. IS $\vec{v} = \langle \frac{1}{2}, \frac{1}{4}, \frac{1}{4} \rangle$ A UNIT VECTOR? **NO**

$$\|\vec{v}\| = \sqrt{\left(\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{4}\right)^2 + \left(\tfrac{1}{4}\right)^2} = \sqrt{\tfrac{1}{4} + \tfrac{1}{16} + \tfrac{1}{16}} = \sqrt{\tfrac{4+1+1}{16}} = \frac{\sqrt{6}}{4} \neq 1$$

IF $\vec{v} = \langle \frac{1}{2}, \frac{1}{4}, k \rangle$ IS A UNIT VECTOR, WHAT IS k?

$$\|\vec{v}\| = \sqrt{\tfrac{1}{4} + \tfrac{1}{16} + k^2} = 1$$

$$\frac{5}{16} + k^2 = 1$$

$$k^2 = \frac{11}{16}$$

$$k = \pm\frac{\sqrt{11}}{4}$$

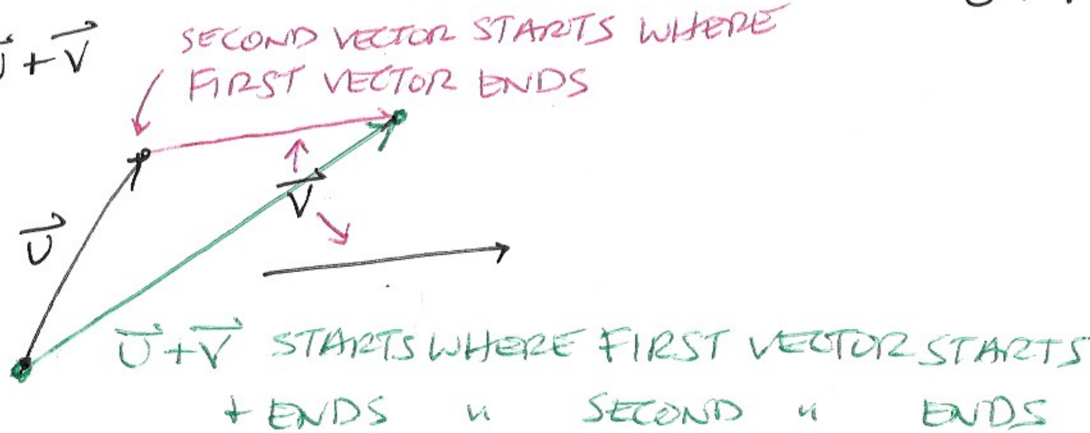
VECTOR OPERATIONS

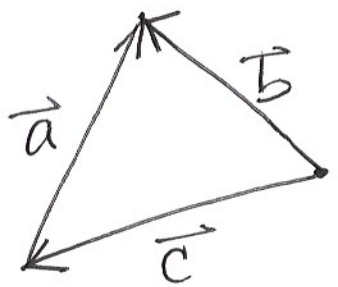
IF $\vec{u}, \vec{v}$ ARE VECTORS

VECTOR ADDITION

$$\vec{u} + \vec{v} = \vec{v} + \vec{u}$$

$\vec{u} + \vec{v}$





$\vec{c} + \vec{a} = \vec{b}$

TERMINAL POINT OF $\vec{c}$
= INITIAL POINT OF $\vec{a}$

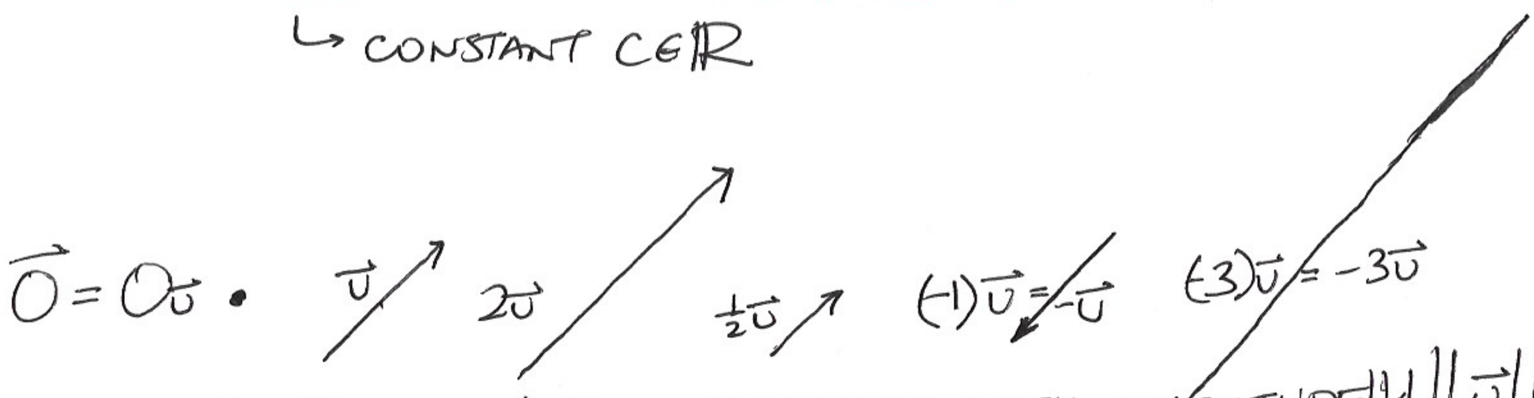
IF $\vec{u} = \langle u_1, u_2, u_3 \rangle$ AND $\vec{v} = \langle v_1, v_2, v_3 \rangle$

THEN $\vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$

eg. $\langle 5, -2, 4 \rangle + \langle -6, 7, 1 \rangle = \langle 5 + -6, -2 + 7, 4 + 1 \rangle$
$= \langle -1, 5, 5 \rangle$

SCALAR MULTIPLICATION

↳ CONSTANT $c \in \mathbb{R}$

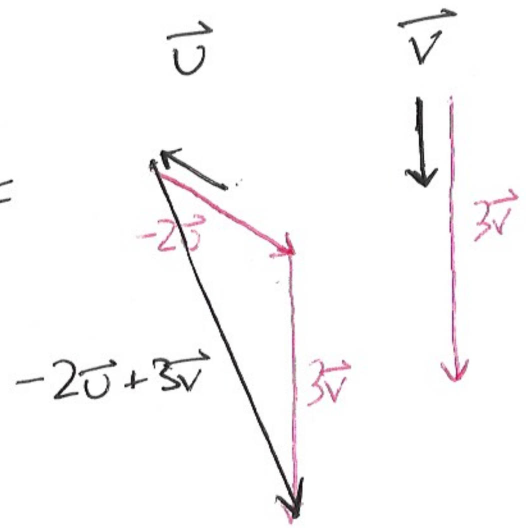


IF $k \in \mathbb{R}$, THEN $k\vec{u}$ IS THE VECTOR WITH MAGNITUDE $|k|\,\|\vec{u}\|$
AND THE SAME DIRECTION AS $\vec{u}$ IF $k > 0$
OPPOSITE IF $k < 0$

$\|k\vec{u}\| = |k|\,\|\vec{u}\|$

SKETCH

$-2\vec{u}+3\vec{v}$ IF



IF $\vec{u}=\langle u_1, u_2, u_3\rangle$

THEN $k\vec{u}=\langle ku_1, ku_2, ku_3\rangle$

eg. IF $\vec{s}=\langle -2,-1,4\rangle$ AND $\vec{t}=\langle 1,-3,2\rangle$

THEN $-5\vec{s}+2\vec{t}=-5\langle -2,-1,4\rangle+2\langle 1,-3,2\rangle$

$=\langle 10,5,-20\rangle+\langle 2,-6,4\rangle$

$=\langle 12,-1,-16\rangle$

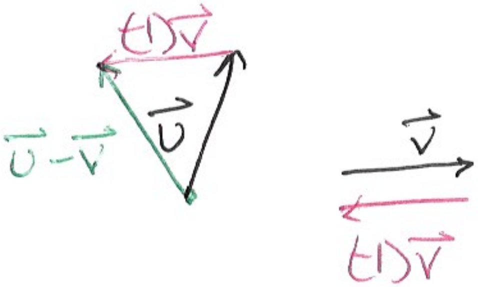
$c_1\vec{u}+c_2\vec{v}$ IS CALLED A LINEAR COMBINATION OF $\vec{u},\vec{v}$

WHAT IS $\vec{u}-\vec{v}$?

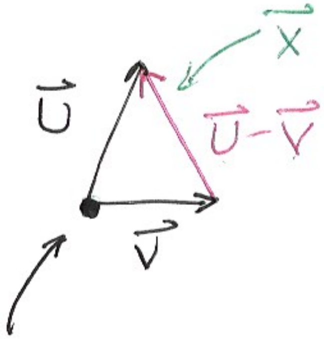
$= \vec{u} + (-1)\vec{v}$

IF $\vec{u} = \langle u_1, u_2, u_3 \rangle$

AND $\vec{v} = \langle v_1, v_2, v_3 \rangle$



OR



$\vec{v} + \vec{x} = \vec{u}$

$\vec{x} = \vec{u} - \vec{v}$

$\vec{v} - \vec{u} = -(\vec{u} - \vec{v})$

SAME INITIAL POINT

$\vec{u} - \vec{v}$ = VECTOR FROM $\vec{v}$'S TERMINAL POINT
TO $\vec{u}$'S TERMINAL POINT
WHEN $\vec{u}, \vec{v}$ HAVE INITIAL POINT

$$\vec{u}+\vec{v}=\vec{v}+\vec{u}$$

$$(\vec{u}+\vec{v})+\vec{w}=\vec{u}+(\vec{v}+\vec{w})$$

$$k(c\vec{u})=(kc)\vec{u}$$

$$c(\vec{u}+\vec{v})=c\vec{u}+c\vec{v}$$

$$(k+c)\vec{u}=k\vec{u}+c\vec{u}$$